Hall Ticket Number:

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Mathematics-I

Time: **3** hours

Max. Marks: **50**

*Note: Answer ALL questions in **Part-A** and any FIVE questions from **Part-B***

Part-A (15 Marks)

1. Examine whether the vectors $u = (1, 1, 1)$, $v = (1, -1, 2)$ and $w = (3, 1, 4)$ are linearly independent. [1]

2. State the p-series test and use it to examine the convergence of the series: $\sum_{n=1}^{\infty} \dfrac{\sqrt{n}}{n^2 + 1}$. [1]

3. Write the formulae for the radius of curvature in (a) Cartesian form and (b) Parametric form. [1]

4. If $u = xe^y z$ and $y = \sqrt{a^2 - x^2}$, $z = sin^2 x$, find $\dfrac{du}{dx}(0,1,1)$. [1]

5. Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dy\, dx$. [1]

6. Define the rank of the matrix and find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. [2]

7. Test for convergence of the series $\sum \left(1 + \dfrac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ [2]

8. Find the Taylor's Series expansion of $f(x) = 2^x$ about x=0. [2]

9. If $u = x^2 - 2y$, $v = x + y$, find $\dfrac{\partial(u,v)}{\partial(x,y)}$. [2]

10. Evaluate $\int\int xy(x + y)$ dx dy over the area between $y = x^2$, $y = x$. [2]

Part-B (5 X 7=35 Marks)

11. a) Find the characteristic equation of matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the inverse of A. [3]

    b) Find the values of 'k' for which the system of equations $(3k - 8)x + 3y + 3z = 0$,

    $3x + (3k - 8)y + 3z = 0, 3x + 3y + (3k - 8)z = 0$ has a non-trivial solution. [4]

12. Test for the convergence of the series :

    a) $\sum_{n=1}^{\infty} \dfrac{n!}{(n^n)^2}$. [3]

    b) $\sum_{n=1}^{\infty} \dfrac{\sqrt{n}}{\sqrt{n^2 + 1}} x^n$. [4]

13. a) Show that the evolute of the curve x=a(cos t + log tan $\frac{t}{2}$), y = a sint is the catenary y = cosh(x/a).   [4]

   b) Find the envelope of the family of straight line $\frac{x}{a} + \frac{y}{b} = 1$, where a,b connected by the relation

   a+b = c, 'c' is a constant.   [3]

14. a) Expand f(x,y) = e^xcos y at (1, $\pi/4$) upto second degree.   [3]

   b) Find the extreme value of the function f(x,y) = $3x^2 - y^2 + x^3$ [4]

15. a) Find the area lying between the parabola y = $4x - x^2$ and the line y = x.   [3]

   b) Find the volume of the region bounded by z = $x^2 + y^2$, z = 0, x = -a, x = a and y = -a, y = a.   [4]

16. a) Reduce $3x^2 + 3z^2 + 4xy + 8xz + 8yz$ into canonical form.   [3]

   b) Test the series for convergence $\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \ldots \ldots \infty$.   [4]

17. Answer any **two** of the following: [7]

   a) Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$.

   b) Find the minimum value of $x^2 + y^2 + z^2$ so that xyz = 8.

   c) Evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y)dx\, dy$ by changing the order of integration.

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